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GROUND COVERAGE OF LIQUID WAR GASES

by

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Summary

The report gives an account of a mathematical model for estimation of the ground coverage as a result of an attack with war gas. The purpose of the model is to furnish an estimate of the size and distribution of the ground coverage following a given hostile effort.

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Introduction

The purpose of this work is to study a model for estimation of the ground coverage after an attack with liquid war gas. For estimation of the extent and intensity of the ground coverage, knowledge of the existing wind and temperature conditions in the strata in question is of great importance. Works concerning the influence of the meteorological factors upon the dispersion are described in bibliographies by Sutton [1] and Pasquill [2]. In Sweden such problems have been dealt with by Froessling [3], [4], [5], Aurivillius [5], Wedin [6], Wickerts [6] and others. In reference 7) is shown a model for estimation of the amount required for an attack with war gas in aerosol form, which has been modified here to make it applicable to gas in liquid form. Reference 7) also gives certain guidelines for estimating the ground coverage obtained during spraying of liquid war gas, but the influence of atmospheric disturbances upon the drops during the spreading is not taken into consideration. In the model proposed here, this factor is taken into account.

1. General Guidelines

There are two main methods of spreading war gases in liquid form: a bomb containing the gas can be detonated by means of a relatively small explosive charge, or the gas can be sprayed from tanks attached to aircraft. The liquid will be in the form of drops of various sizes, most of which will have diameters of between 50 μm and 1,000 μm. Even though the amount by weight present in the form of drops less than 50 μm in diameter is insignificant, this part of the substance is of great importance to poisoning of the atmosphere for a long time after the spreading. In general, the height from which the spreading is done varies between 15 and 100 meters. During the fall of the drops toward the ground, a vertical speed equilibrium (v_z), depending upon drop size and the density of the liquid, is rapidly reached. The speed of the drops in the wind direction may be regarded as equal to the wind speed (v_x). Together with the original position of the drop, v_z and v_x determine the expected path of the drops. Atmospheric turbulence will cause departure from the expected path. Here, we suppose that the drops during their descent spread as if they were gas particles (see Section 53.). The departures of the particles can be described by the dispersion values or σ values obtained from meteorological data. These depend upon existing wind conditions, the descent period of the drops, and upon the type of atmospheric stratificiation. The σ values in the wind direction (σx), at right angle to the wind direction (σy), and vertically (σz) are here assumed to be equal (see Appendix 1). This is a rather exact approximation for indifferent stratification, and often for stable stratification. It is at these types of stratification that a scattering of this kind can be expected to occur, while the effect at a labile stratification might be very hard to judge. The scattering device may also locally increase the atmospheric turbulence.

2. The Primary Cloud

The primary cloud is the designation for that cloud of liquid drops which forms after the spraying. It presents various appearances, depending upon the production method. If the primary cloud is produced by means of a bomb, it is often of spherical or similar shape; if formed by means of spraying from a tank it appears as a long string of relatively little height and width. The primary cloud is assumed to be built up of a number of right-angle parallelepipeds, with sides l_x, l_y, and l_z. These elements are called cloud elements. The distribution of drop sizes within the various cloud elements is in the following assumed to be generally the same in all elements. Each cloud element is thought to be built up of a number of stratified unit elements, each of which contains a narrow size fraction of the drops.

For estimation of the coverage from a primary cloud, the cloud is assumed to be divided into unit elements. The coverage from each unit element is calculated and adding these coverages gives the coverage from the primary cloud (see 61). Programming for a [data processing] machine has been worked out for such calculations; it is shown in Appendix 2. In this case, then, the three dimensions of the primary cloud and the drop size are added up. In certain cases (see 62) integrals can be substituted for these sums, which lessens the time required for the calculations.

3. Distribution of Drop Sizes

Drops of liquid of various sizes are produced by the formation of the primary cloud. The distribution of the drop sizes depends upon the method of delivery. Among the distribution functions available in the literature, the so-called Nukiyama-Tanasawa [8] distribution has been selected here:

$$\frac{dN}{dX} = a.X^p.\exp\left\{-bX^q\right\} \qquad (3.1)$$

In practical cases, it may become necessary to examine pragmatically the validity of this assumption. In particular, it may be important to check the distribution of very small and very large drops, in the former case because of the increased risk of poisoning via the respiratory system, and in the latter case because of the greater mass.

In (3.1) a, b, p, and q are parameters. N represents the relative number (fraction of the total), and X the drop diameter in μm. Thus the fraction of drops to which $X_1 < X \leqslant X_2$ apply is

$$\int_{X_1}^{X_2} dN$$

If parameters p and q are assigned values 2 and 1, respectively, good agreement is found with the empirically found distribution curves of reference 5).

The volume of all drops having a diamter of $\leq X$ becomes:

$$V = \int_{0}^{X} \frac{\pi}{6} \cdot X^3 \cdot dN \qquad (3.2)$$

From (3.1) and (3.2) and with the figure values for p and q there is obtained:

$$\frac{dV}{dX} = \frac{a \cdot \pi}{6} \cdot X^5 \cdot \exp\left\{-bX\right\} \qquad (3.3)$$

The following designations are introduced:

$$V_o = \int_{0}^{\infty} dV \qquad (3.4)$$

in which V_o designates the total spray volume.

The various central values in question for this distribution are obtained as follows:

$$\frac{d^2V}{dX^2} = 0 \qquad (3.5)$$

gives $X = D$, where D designates the maximum value or the type value (volume-mode-diameter) for the frequency function $\frac{dV}{dX}$

$\int_{0}^{D_m} dV = 0.5 \cdot V_o$ gives the value D_m which is (3.6)

the (volume-) median diameter

The parameters a and b can now be expressed in V_o and D as follows:

$$a = \frac{6.b^6.V_o}{\pi .5!} \qquad (3.7)$$

$$b = \frac{5}{D} \qquad (3.8)$$

Further, there is the following connection between D and D_m when p = 2 and q = 1:

$$D_m = D.1.1340$$

which is obtained from (3.3), (3.6), (3.7) and (3.8).

If (3.7) and (3.8) are introduced into (3.3) there is obtained

$$\frac{dV}{dX} = \frac{5^5.V_o}{24.D^6} \; X^5.\exp\left\{-\frac{5}{X}X\right\} \qquad (3.9)$$

In direct spraying from aircraft there is obtained, by means of a formula according to Merrington and Richardson [9], D_m in μm from the relation

$$D_m = \frac{0.57.10^7.\sqrt[5]{\nu}}{v_a} \qquad (3.10)$$

in which ν = kinematic viscosity in Stokes

v_a = relative speed between the liquid jet and and the ambient air, cm/sec.

For devices tested in aircraft experiments by Froessling and Aurivilius [5] there was obtained

$$D_m \approx \frac{10^7 . \sqrt[5]{\nu}}{v_a} \qquad (3.11)$$

In the division of a cloud element into unit elements, (3.9) is approximated with a discrete frequency function. The X axis is divided tino m number of intervals of equal size. The intervals become (i-1).b' to i b' where i assumes the values 1, 2 . . . m. In the discrete frequency function, X can only assume the value corresponding to the m number of the middle points of the intervals, thus $X_1 = .5.b'$, $X_2 = 1.5 . b'$, $X_k = (k - .5). b'$. . . $X_m = (m - .5). b'$. In the interval around X_k the spread volume becomes

$$\int_{(k\ \ 1)\ .\ b'}^{k\ .\ b'} dV$$

m is selected, for example, so that

$$\int_{o}^{m\ \ b} dV \geq 0.999.V_o$$

4. The Speed of Descent of the Drops

The speed of descent (v_z) of the drops depends on their diameters and density. Here, as in reference 7), the following relatinn between drop diameter (μm) and speed of descent (m/s) is used:

$$v_z = k.\ X^{[C_1 + C_2 \log X + C_3 (\log X)^2]} \qquad (4.1)$$

in which k = Stokes' constant - 3.0044 . 10^{-5} in air at 70°F (21.1°C) for a drop density of 1 g/cm^3.

For drops of $X < 53\ \mu$m, $C_1 = 2$ and $C_2 = 0$.

For drops in the interval $53 < X < 800\ \mu$m, $C_1 = 2.01302$, $C_2 = .12055$, and $C_3 = -.07422$ [sic].

Since the number of drops of diameter $> 800\ \mu$m are probabl very rare, (4.1), with the latter values for the constants, has been used in the area X 800 m. Exact values may be obtained, for example, according to Froessling [10].

It is assumed here that evaporation from the drops is negligible. When this is not the case, however, the calculations can be based on the actual time of descent.

5. The Spread of the Unit Element and Coverage From It

51. Designations (see Fig 1)

x, 6, and z designate the positions in a right angle coordinate system with the x-axis in the wind direction. The positive z-axis designates height above the ground. The y-axis is at a right angle to the x-z plane.

$(X_1 ; y_1)$ position in the ground plane.

t variable time. $t=0$ is time of formation of the cloud

t_s point of time when protection is obtained

r distance from a point to the center of the cloud

C concentration

C_m concentration at the center of the cloud

$(x_m ; y_m ; z_m)$ position of the center of the cloud

$(x_o ; y_o ; z_o)$ positions of the center points at $t = 0$

r_o distance from the center of the cloud to a point where the concentration is $.05 \cdot C_m$

t_{o1} point in time when the distance from a point (x_1 , y_1) to the center of the cloud is minimal

r_{o1} is r_o at point in time t_{o1}

$\bar{v}_x$ median wind speed (see Appendix 1)

v_z speed of descent

w the amount of matter in the cloud

Δ_x, Δ_y, Δ_z constants for the turbulent diffisuion in x-, 6- and z- links

σ_x, σ_y, σ_z the spread of drop around its expected path in x-, y- and z- links

σ the spread of a drop about the expected path of the center of the cloud

n layering constant

$D(x_1 ; y_1)$ dosage at the point $(x_1 ; y_1)$ up to the time t_s

$B(x_1 ; y_1)$ coverage at the point $(x_1 ; y_1)$ up to the time t_s

52. The Development of the Unit Elements

At the point of time $t = 0$ the unit element has the sides l_x, l_y and l_z and contains drops of so narrow a size fraction that the drop size may be regarded as constant. The concentration at all points of the cloud is assumed to be the same at the point of time $t = 0$. Thus, if one regards the concentration distributions in x, y, or z links there is obtained a rectangular concentration distribution with the variation widths l_x, l_y, l_z, respectively. The variance for such a distribution is $l^2/2$. Immediately after the point of time $t = 0$ the particles become exposed to the disturbance caused by atmospheric turbulence. Due to this disturbance, each drop deviates randomly from its expected path. The deviations from the expected path in both x, y, and z links are regarded as normally distributed. Thus, the distribution of the particles around the path of the center of gravity of the cloud will be composed of a rectangular distribution (the original position of the drop in the unit element) and a normal distribution (atmospheric disturbance). The spread of the normal distribution, which grows with time, after a rather short time becomes large in relation to the constant spread of the rectangular distribution. The concentration of the cloud can then be

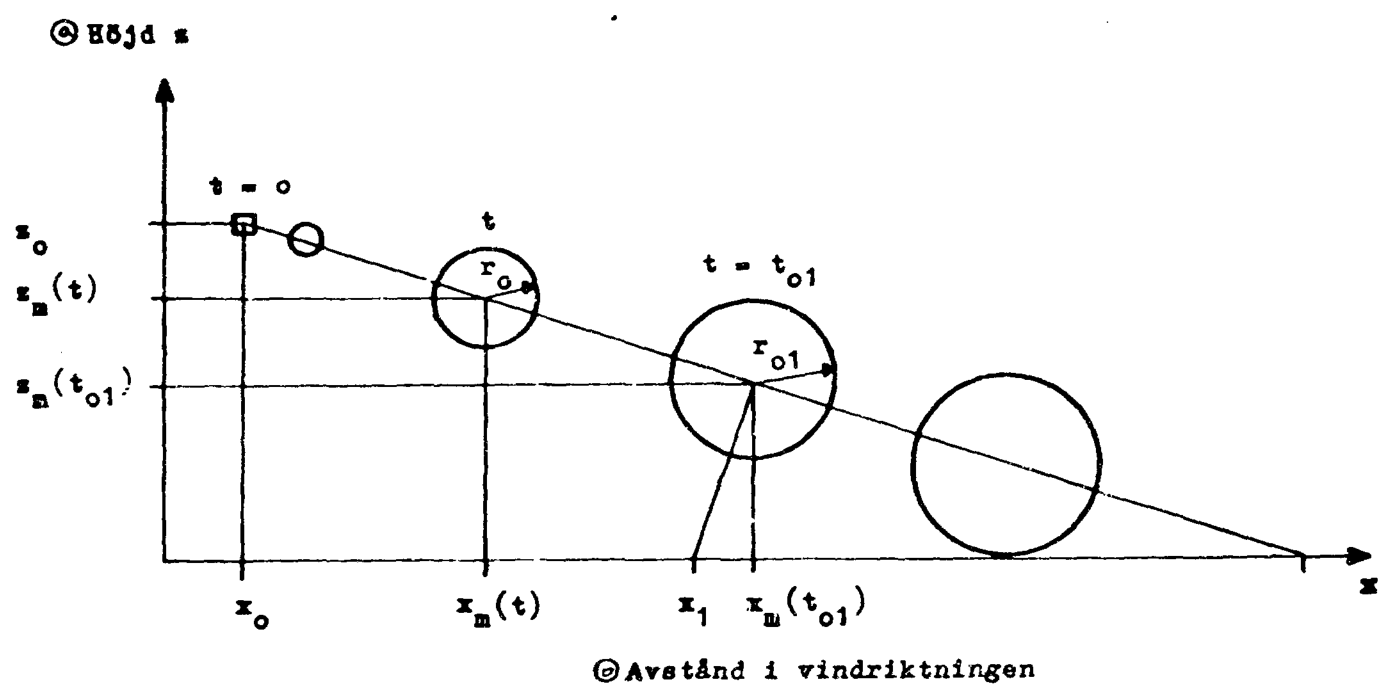

Figure 1. Schematic Representation of the Development of a Unit Element

/Legend/: (a) Height; (b) Distance in wind direction.

regarded as determined by a normal distribution, in which the spread is composed of the spread of the two distribution.

As previously mentioned, the expected path of the center of gravity of the unit element is determined by the original position of the center of gravity and wind speed and speed of descent. The wind speed usually increases with the altitude (the wind speed profile), and the path therefore is not a straight line. At the altitudes we are concerned with here the spread values have been assumed to be independent of the altitude. Further, it has been assumed that a designed so-called equivalent wind speed, $\bar{v}_x$, a constant in the strata in question, can be substituted for the wind speed increase with altitude (see further Appendix 1). This wind speed equivalent, $\bar{v}_x$, depends for a certain ground wind speed and layer constant upon the original altitude of the unit element. The wind equivalent is so selected that the expected point of impact of the center of gravity is the same as if the real wind speed profile had been used. In accordance with this simplification the path of the center of gravity becomes a straight line. Thus the center of gravity is assumed to be displaced at the constant speed $\bar{v}_x$ in the wind direction, while at the same time it falls at the constant speed v_z. The displacement $x_1 - x_0$ in the wind direction during the fall from the original altitude z_0 becomes

$$x_1 - x_0 = \frac{v_x}{v_z} \cdot z_0 \quad .$$

53. Calculation of the Ground Coverage From a Unit Element

The unit element is considered to become a cloud with the concentration gradient determined by the normal distribution long before the ground level is affected by the cloud. It can, therefore, be assumed that the unit element immediately turns into a normal distribution cloud with the variances $l^2{}_x/12$, $l^2{}_y/12$, and $l^2{}_z/12$ in the x-, y-, and z-links, respectively. After the point of time t = 0 the spread of the cloud caused by atmospheric disturbances begins. The variance of gas element around its expected path due to this cause is, according to Sutton (see Appendix 1);

$$\sigma_x^2 = 0.5 \cdot \Delta^2_x \cdot (t \cdot \overline{v}_x)^{2-n}$$

$$\sigma_y^2 = 0.5 \cdot \Delta^2_y \cdot (t \cdot \overline{v}_x)^{2-n} \qquad (5.1)$$

$$\sigma_z^2 = 0.5 \cdot \Delta^2_z \cdot (t \cdot v_x)^{2-n}$$

For altitudes of <100 m it is often approximately holds true for stable and indifferent stratification that:

$$\Delta x = \Delta y = \Delta_z = \Delta$$

and therefore

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2$$

It is here assumed that the drops during their descent have the same deviations as gas elements from their expected paths, so that the σ values according to (5.1) are valid regardless of the drop size (W. Schmidt [11]). For large drops, however, diminished spread may occur for two reasons: the restraining effect of inertia, and because the drops enter new urbulence more quickly because of their greater speed, (see Pasquill [2], pp 112, 226). In certain cases of calculations according to the formula below it may be necessary to employ σ values, which in this respect vary with drop sizes.

If one selects $l_x = l_y = l_z = l$ the variance of the drops around the expected path of the center of gravity for the three main directions becomes:

$$\sigma^2 = 0.5 \; \Delta^2 \cdot (t \cdot \overline{v}_x)^{2-n} \quad l^2/12 \qquad (5.2)$$

Since the variances in the main directions are assumed to be equal, points of the same concentration are located on a spherical shell. At the distance r from the middle point of the cloud this concentration is

$$C(r) = C_m \cdot \exp \left\{-r^2/2\,\sigma^2\right\}$$

Here $C(r_o) - 0.05 \cdot C_m$

$$0.05\ C_m = C_m \cdot \exp\left\{-r_o^{\ 2}/2\sigma^2\right\}$$

which gives

$$r_o^2 = 6.0 \cdot \sigma^2 \tag{5.3}$$

Thus

$$C(r) = C_m \cdot \exp\left\{-3r^2/r_o^2\right\} \tag{5.4}$$

C_m can be determined from the amount of substance in the unit element.

$$w = 4 \cdot \int_0^\infty r^2 \cdot C(r) \cdot dr$$

which gives

$$C_m = \frac{w \cdot 3 \cdot \sqrt{3}}{\pi \cdot \sqrt{\pi} \cdot r_o^3} \tag{5.5}$$

If (5.5) is introduced into (5.4) there is obtained

$$C(r) = \frac{w \cdot 3 \cdot \sqrt{3}}{\pi \cdot \sqrt{\pi} \cdot r_o^3} \cdot \exp\left\{-3r^2/r_o^2\right\} \tag{5.6}$$

The position of the middle point at time t is determined by the following equations:

$$x_m = x_o + t \cdot \bar{v}_x \tag{5.7a}$$

$$y_m = y_o \tag{5.7b}$$

$$z_m = z_o - t \cdot v_z \tag{5.7c}$$

The distance from the points (x_1 ; y_1) to the middle point of the cloud can be written:

$$r^2 = (x_1 - x_m)^2 \quad (y_1 - y_m)^2 \quad z_m^2 \tag{5.8}$$

If equations (5.7) are introduced into (5.8), r^2 is obtained as a function of the time

$$r^2 = (x_1 - x_o - t \cdot \bar{v}_x)^2 + (y_1 - y_o)^2 + (z_o - t \cdot v_z)^2 \quad (5.9)$$

If (5.9) is introduced in (5.6) the concentration in points $(x_1 ; y_1)$ is obtained as a function of the time.

$$C(t) = \frac{w \cdot 3 \cdot \sqrt{3}}{\pi \cdot \sqrt{\pi} \cdot r_o^3} \cdot \exp\left\{- \frac{3}{r_o^2}\left[(x_1 - x_o - t \cdot \bar{v}_x)^2 + (y_1 - y_o)^2 + (z_o - t\, v_z)^2\right]\right\} \quad (5.10)$$

For calculation of the dosage and the coverage a simplification is introduced: the cloud is assumed to be of constant size during passage of the points $(x_1 ; y_1)$. During this passage, the cloud is assumed to have the C_m and r_o values which exist when the distance from the points to middle point of the cloud is minimal. (Before and after this point the cloud is in reality smaller and larger, respectively.) The minimal distance occurs at time t_{o1}. The radius of the cloud is then $r_{o1} = r_o\,(t_{o1}) \cdot t$ is obtained through reducing (5.9), which results in

$$t_{o1} = \frac{v_x(x_1 - x_o) + v_z \cdot z_o}{\bar{v}_x^2 + v_z^2} \quad (5.11)$$

Introduction of t_{o1} in (5.2) and (5.3) gives:

$$r_{o1}^2 = 3 \cdot \Delta^2 \cdot (t_{o1}\, \bar{v}_x)^{2-n} + 0.5 \cdot l^2$$

If 1 m is selected for the size for l, the last term can be eliminated. The dosage in points $(x_1 ; y_1)$ is obtained from:

$$D(x_1 ; y_1) = \int_0^{t_s} C(t) \cdot dt$$

If (5.10) is introduced, there is obtained:

$$D(x_1 ; y_1) = \frac{w \cdot 3 \cdot \sqrt{3} \exp\left\{- \frac{3}{r_{o1}^2} (y_1 - y_o)^2\right\}}{\pi \cdot \sqrt{\pi} \cdot r_{o1}^3} \cdot$$

$$\int_0^{t_s} \exp\left\{- \frac{3}{r_{o1}^2} \cdot \left[(x_1 - x_o - t \cdot \bar{v}_x)^2 + (t . v_z - z_o)^2\right]\right\} dt \quad (5.12)$$

Through the substitutions

$$E = \bar{v}_x (x_1 - x_o) + v_z \cdot z_o$$

$$A^2 = v_x^2 + v_z^2$$

and the transformation

$$\frac{3}{r_{o1}^2} (At - \frac{E}{A})^2 = \frac{u^2}{2}$$

there is obtained

$$D(x_1 ; y_1) = \frac{w \cdot 3 \cdot \exp\left\{- \frac{3}{r_{o1}^2} \left[(y_1 - y_o)^2 + (x_1 - x_o)^2 + z_o^2 - \frac{E^2}{A^2}\right]\right\}}{r_{o1}^2 \cdot A \cdot \pi}$$

$$\int_a^b \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{u^2}{2}} \cdot du \quad (5.13)$$

where the integration limits b and a are defined by:

$$b = \frac{\sqrt{6}}{r_{o1}} (A \cdot t_s - \frac{E}{A}) \qquad a = - \frac{\sqrt{6} \cdot E}{r_{o1} \cdot A}$$

The coverage (quantity/m^2) is in simple relation to the dosage (time · quantity/m^3), which is apparent from the following: The additional quantity ΔB during the time interval t for a point at ground level derives from drops found between ground level and the height $\Delta t \cdot v_2$ above ground level. The concentration during the time interval Δt in the area of the point is constant and equals $C(t)$.

The additional quantity can, therefore, be expressed:

$$\Delta B = \Delta t \cdot v_z \cdot C(t)$$

After Limes transition (to differentials) and integration there is obtained:

$$B(x_1 ; y_1) - v_z \cdot \int_0^{t_s} C(t)\, dt = v_z \cdot D(x_1 ; y_1)$$

The coverage can thus be obtained from the dosage by multiplication by the speed of descent.

54. Summary of Formulas

$$E = \bar{v}_x (x_1 - x_o) + v_z \cdot z_o$$

$$A^2 = v_x{}^2 + v_z{}^2$$

$$t_{o1} = \frac{\bar{v}_x (x_1 - x_o) + v_z \cdot z_o}{\bar{v}_x^2 + v_z} = \frac{E}{A^2}$$

$$r_{o1}{}^2 = 3 \cdot \Delta^2 \cdot (t_{o1} \cdot v_x)^{2-n} + 0.5 \cdot 1^2$$

$$D(x_1 ; y_1) = \frac{3 \cdot w \cdot \exp - \left\{ \frac{3}{r_{o1}{}^2} \left[(y_1 - y_o)^2 + (x_1 - x_o)^2 + z_o^2 - \frac{E^2}{A^2} \right] \right\}}{\pi \cdot A \cdot r_{o1}{}^2}.$$

$$\int_a^b \frac{1}{\sqrt{2}} \cdot e^{\frac{-u^2}{2}} \cdot du$$

$$a = - \frac{\sqrt{6} \cdot E}{r_{o1} \cdot A}$$

$$b = \frac{\sqrt{6}}{r_{o1}} \left(A\ t_s - \frac{E}{A}\right)$$

$$B\ (x_1\ ;\ y_1) = v_z \cdot D(x_1\ ;\ y_1)$$

6. Calculation of the Coverage From a Primary Cloud

61. Addition of Unit Elements.

The primary cloud is described by the positions and contents of the cloud elements. The middle point of the i-te cloud element is designated $(x_{oi}; y_{oi}; z_{oi})$. The total amount of substance in the primary cloud is designated W. The amount of substance in unit element j of the i cloud element is designated w_{ij}. Therefore:

$$\sum_j w_{ij} = w_i$$

$$\sum_i \sum_j w_{ij} = \sum_i w_i = W .$$

in which w_i designates the amount of substance in the i-te cloud element.

The drop size distribution within the various cloud elements is assumed to be the same. In the preceding section the coverage from a unit element was calculated. The coverage from a primary cloud is obtained through addition of the coverages from all of the unit elements. These calculations are carried out by means of computer. A program for the computer is shown in Appendix 2.

In dealing with a primary cloud originating in spraying from a tank attached to an aircraft, the primary cloud can with adequate exactness be described as a number of cloud elements lying above one another and with their central points lying in a straight line in space. The size, form, and position of the primary cloud is thus determined by the position of the first cloud element (x_{o1} ; y_{o1}; z_{o1}), the number of cloud elements, M, and the position of the last cloud element (x_{oM}; y_{oM}; z_{oM}). The position of i cloud element is then (x_{oi}; y_{oi}; z_{oi}) in which

$$x_{oi} = x_{oi} + (i-1)\frac{x_{oM} - x_{o1}}{M-1}$$

and the corresponding expressions for y_{oi} and z_{oi}.

Figures 2 shows the ground coverage calculated for a unit element, produced at an altitude of 25 m, having sides of 1 m, with wind speed at 7 m/s, and the layering constant, n, at .35, i.e., Δ=.07 (see table in Appendix 1). The substance content of the unit element is 1 g and the substance density 1.

Figure 3 shows under the same conditions B(x; 0), i.e. the coverage on the x axis, from unit elements corresponding to two different drop sizes.

Figure 4 shows the ground coverage from a primary cloud sprayed at right angles to the wind direction at the altitude of 25 m. The cloud has a length of 1,000 m and is regarded as being built up of a row of cloud elements, each of which has sides of 1 m. Wind speed, layering constant, and substance density as in Figure 1. The cloud has a drop size distribution according to (3.9), with the median diameter at 280 μm. The substance content is 300 g/m /sic; should probably read 300 g/m²/. In the calculation of the coverage the cloud elements were divided into 17 unit elements, each of them including a drop size interval of 50 μm. An investigation of the effect of interval length upon the coverage has shown that a finer division than 50 μm is unnecessary. The substance content and the speed of descent for the various unit elements are shown in the table below. The speed of descent of a unit element is calculated from (4.1), in which the drop size, X, is equal to the middle point of the interval.

Drop size for the unit cube (μm)	Substance content, w(g)	Speed of descent v_z (m/s)
0 - 50	0.190	0.019
50 - 100	5.055	0.154
100 - 150	21.079	0.350
150 - 200	40.496	0.578
200 - 250	51.682	0.820
250 - 300	51.386	1.067
300 - 350	43.211	1.314
350 - 400	32.228	1.557
400 - 450	21.966	1.794
450 - 500	13.958	2.025
500 - 550	8.385	2.247
550 - 600	4.812	2.462
600 - 650	2.658	2.668
650 - 700	1.421	2.866
700 - 750	0.739	3.055
750 - 800	0.375	3.237
800 - 850	0.360	3.411

In this case, where the dissemination was done at right angle to the wind direction and at a constant altitude, it was possible to introduce certain simplifications into the computer program. As indicated in the foregoing and as is apparent from Appendix 2, one can by means of the computer program worked out obtain for an arbitrary point on the ground level, the coverage from a primary cloud element consisting of a cloud element in an arbitrary configuration in space. By means of a minor adjustment to the computer program, it is in addition possible to vary the drop size distribution from cloud element to cloud element.

62. Primary Cloud Integration

In 61 the coverage at a point (x_1 ; y_1) was calculated by summation of (5.13) on drop size X and on the dimensions x_o, y_o and z_o of the primary cloud. The total coverage at (x_1 ; y_1) can thus be written

$$\sum_{X} \sum_{x_o} \sum_{y_o} \sum_{z_o} v_z \cdot D(x_1 \; ; \; y_1)$$

If the form of the primary cloud can be given in a simple mathematical expression, one or more of the symbols of the above formula can to advantage be replaced by integrals. Froessling [4] has shown such calculations for gas clouds.

Here we shall only show that the primary cloud can be regarded as a linear cloud at altitude z_o, where the angle between the direction of spraying and the wind direction is determined by the constant k in relation $x_o = k \cdot y_o$.

The coverage at (x_1 ; y_1) is now calculated from:

$$B(x_1 \; ; \; y_1) = \sum_{X} v_z \int_{y_o} D(x_1 \; ; \; y_1) \, dy_o$$

$\int_{y_o} D(x_1 \; ; \; y_1) \cdot dy_o$ is the dosage originating from y_o drops of a certain size, giving the speed of descent v_z. This integral will probably be of interest in many applications, and the solution will therefore be shown.

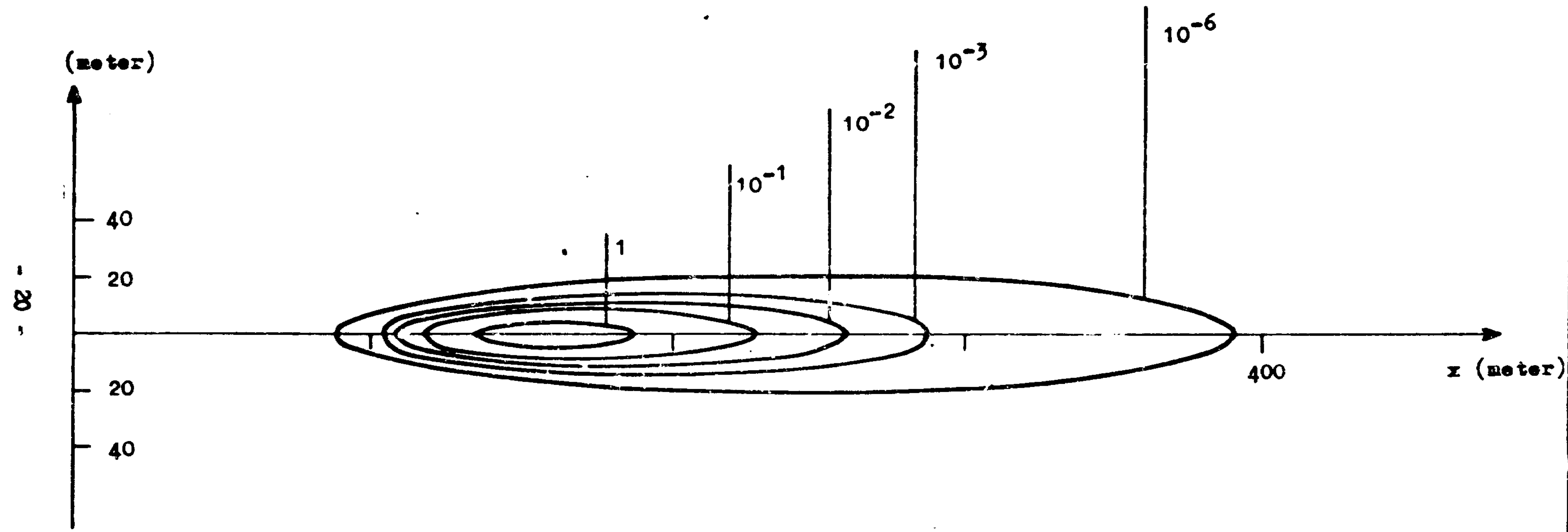

Figure 2. Isocoverage curves.

The curves show the coverage (mg/m^2) after the passage of a unit element produced at an altitude of 25 m, which contains 1 g of substance of the of the 250-300 m fraction. Wind speed (at 5 m altitude) is 6 m/s, n = .35, and Δ = .07. Substance density is 1.

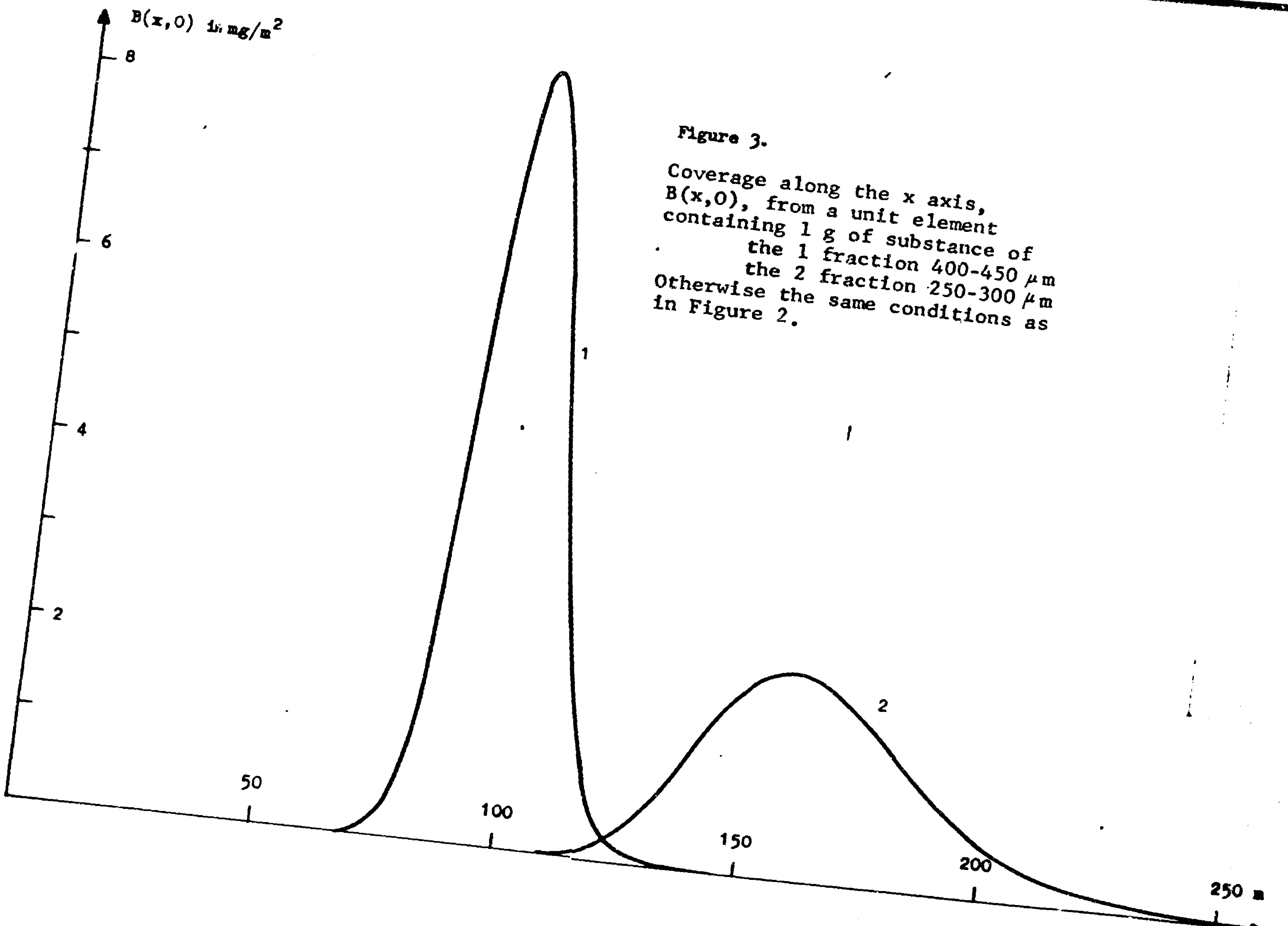

Figure 3.

Coverage along the x axis, B(x,0), from a unit element containing 1 g of substance of
the 1 fraction 400-450 μm
the 2 fraction 250-300 μm
Otherwise the same conditions as in Figure 2.

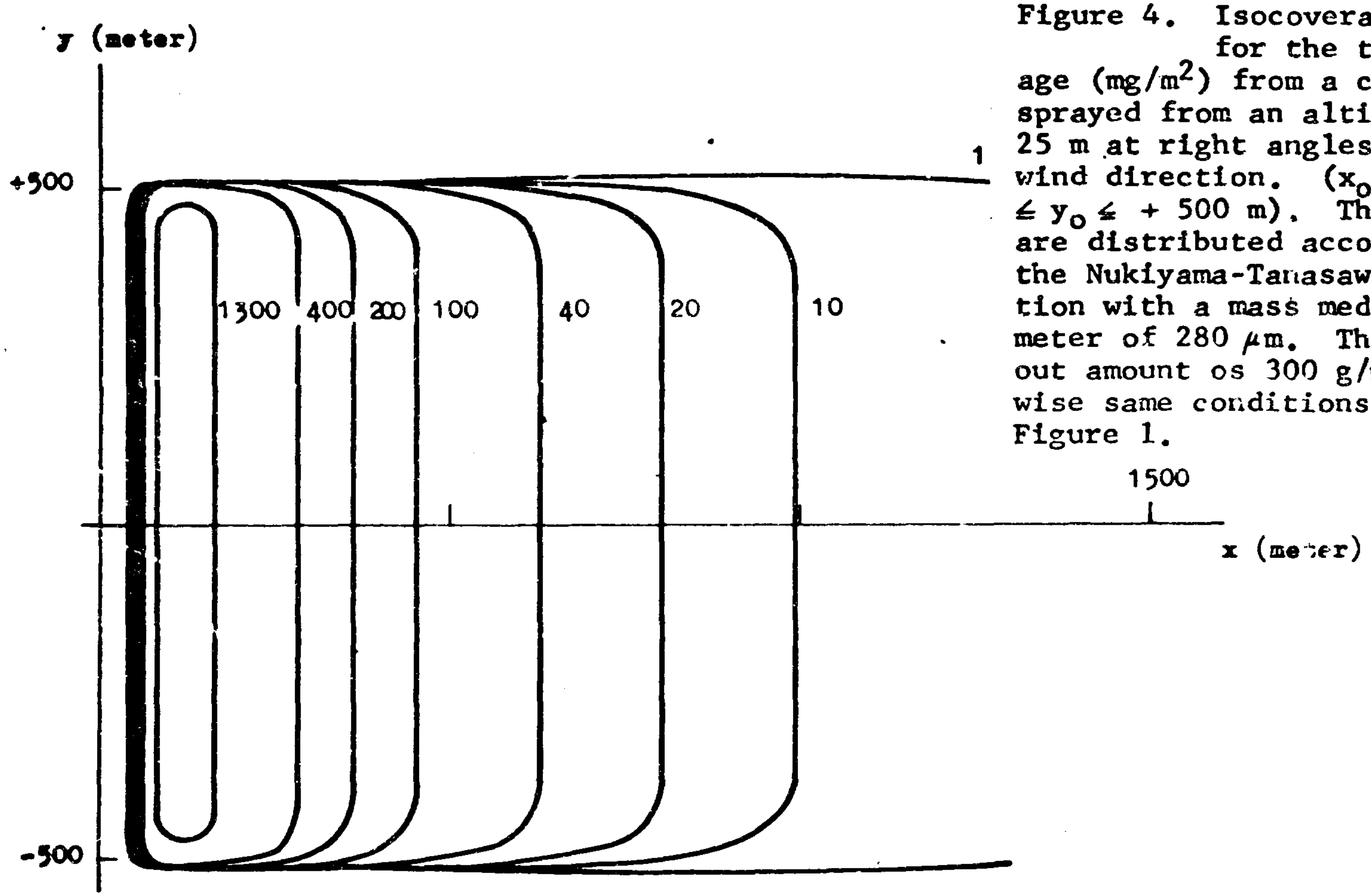

Figure 4. Isocoverage curves for the total coverage (mg/m^2) from a cloud sprayed from an altitude of 25 m at right angles to the wind direction. (x_o-0, $-500 \leq y_o \leq +500$ m). The drops are distributed according to the Nukiyama-Tanasawa distribution with a mass median diameter of 280 μm. The sprayed out amount os 300 g/m. Otherwise same conditions as in Figure 1.

$$\int_{y_o} D(x_1 \, ; \, y_1) \, dy_o =$$

$$= \frac{w_y \cdot \sqrt{3}}{r_{o1} \cdot A \cdot \sqrt{\pi} \cdot \sqrt{\alpha}} \cdot \exp \left\{ - \frac{3}{r_{o1}^2} \left[(x_1^2 + y_1^2 + z_o^2) - \frac{1}{A^2} \right. \right.$$

$$\left. \left. (v_x \cdot x_1 + v_z \cdot z_o)^2 - \frac{\beta}{\alpha}^2 \right] \right\} \cdot$$

$$\int_{q_1}^{q_2} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{q^2}{2}} \left[\int_a^b \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{u^2}{2}} \cdot du \right] \cdot dq \qquad (6.1)$$

where w_y is the amount of substance per unit of length in the y link. w_y is calculated from the amount of substance per unit of length in the cloud, w_1, with the relation

$$w_y = w_1 \cdot \sqrt{1 + k^2}$$

$$\alpha = k^2 + 1 - \frac{k^2 \, \bar{v}_x^2}{A^2}$$

$$\beta = x_1 \cdot k + y_1 - \frac{k \, \bar{v}_x \, (\bar{v}_x \cdot x_1 - v_z \cdot z_o)}{A^2}$$

$$q = \frac{\sqrt{6}}{r_{o1}} \cdot (y_o \cdot \sqrt{\alpha} - \frac{\beta}{\sqrt{\alpha}})$$

r_{o1} is taken from the formulas for r_{o1} and t_{o1}, page 15, with $x_o = k \cdot y_1$. This relation is used also in calculating the integration limits a and b.

From the last formula, the integration limits q_1 and q_2 are calculated from the corresponding limits for y_o.

In formula (6.1) it is possible to remove the parentheses from the integral and to give it a median value.

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APPENDIX 1

THE EFFECT OF METEOROLOGICAL FACTORS UPON THE MODEL

Bo Rydgren, Staff Meteorologist

To make the calculation method described in the foregoing applicable in the simplest way, the wind speed v_x, variable with altitude, has been replaced by a so-called "equivalent wind," $\bar{v}_x$, which gives the middle points of the individual drops the same point of impact as if v_x were used.

$\bar{v}_x$ is obtained from

$$\bar{v}_x \cdot t_1 = \int_0^{t_1} v_x \cdot dt \qquad \text{(B1.1)}$$

in which t_1 is the time from the moment of release to the time the particles reach the ground.

According to Deacon [12] the real wind's dependence on the altitude can be written

$$v_x(z) = k \cdot z^{\frac{n}{2-n}} \qquad \text{(B1.2)}$$

in which z is the altitude at which the wind speed is v_x, n the layering constant (see below), and k a constant, the value of which can be determined from a wind speed measured at a certain altitude. If for example, the wind at the altitude of 5 m, v(5) is assumed known, v_x is obtained from (B1.2) as follows

$$\log v_x(z) = \frac{n}{2-n} (\log z - \log 5) + \log v_x(5) \qquad (B1.3)$$

As is seen, wind direction variation with altitude has not been dealt with, although in certain cases it may have a considerable effect upon the spreading (due to convection, ground friction, or in connection with speed of descent of the drops, for example).

If the speed of descent of the drops is assumed to be constant, (B1.1) can be written:

$$\bar{v}_x \cdot z_o = \int_o^{z_o} v_x \cdot dz \qquad (B1.4)$$

in which z_o is the release altitude.

From (B1.2) and (B1.4) there is obtained:

$$\bar{v}_x = z_o \left| \frac{k.z^{\frac{2}{2-n}}}{\frac{2}{2-n}} \right|_o^{z_o} = k. \frac{2-n}{2} \cdot z_o^{\frac{n}{2-n}} \qquad (B1.5)$$

Finally, from (B1.2) and (B1.5) there is obtained for the altitude z_h with a calculated k value for the measured wind speed $v(z_h)$:

$$\bar{v}_x = (1 - \frac{n}{2}) \cdot v_x(z_h) \cdot \left(\frac{z_o}{z_h} \right)^{\frac{n}{2-n}} \qquad (B1.6)$$

From (B1.6), $\bar{v}_x$ can thus be calculated for a given layering, represented by the constant n (see below) and for known wind speed $v_x(z_h)$, measured at the altitude z_h (see equation B1.3).

It should be added that the above reasoning can be applied only for altitudes up to about 100 m, and that the wind speeds calculated from (B1.2) should be regarded only as probable values, from which great deviations may occur in certain cases.

According to Sutton[1)], the following connections can be used for determination of the time dependency for the variance in the particle distribution within the individual drop cloud:

$$\sigma_x^2 = 0.5\ \Delta_x^2\ (\bar{v}_x \cdot t)^{2-n}$$

$$\sigma_y^2 = 0.5\ \Delta_y^2\ (\bar{v}_x \cdot t)^{2-n} \qquad (B1.7)$$

$$\sigma_z^2 = 0.5\ \Delta_z^2\ (\bar{v}_x \cdot t)^{2-n}$$

where the diffision constants Δ_x, Δ_y, and Δ_z have been determined for various wind speeds in field experiments. In this case, the previously definea "equivalent wind" $\tilde{v}_x$ was used.

It can be shown that at indifferent, and often at stable layering, within 0 - 100 m the following is valid

$$\Delta_x \sim \Delta_y \sim \Delta_z = \Delta$$

For calculation of the successive spread of the droplet cloud from the moment of release until contact with the ground the Δ values according to the table below, calculated for various layering conditions and wind speeds [13)], are used.

Table. Numerical Δ values for various values of v and n, applicable to slightly rolling terrain and spread times ≤ 15 min.

(1) Typ av skikt-ning	(2) Skikt-nings-konstant n	(3) vindhastighet, v_x m/s									
		1	2	3	4	5	6	7	8	9	10
labil	< 0,20	0,52	0,33	0,28	0,23	0,20	0,18	0,15	x-	-	-
neutral	0,20-0,30	0,21	0,19	0,17	0,15	0,13	0,12	0,11	0,11	0,12	0,13
stabil	0,30-0,40	0,10	0,09	0,08	0,07	0,06	0,07	0,07	0,07	0,08	0,08
(4) mycket stabil xx	> 0,40	0,05	0,04	0,03	0,02	x-	-	-	-	-	-

x Probably constant in the continuation, with the wind within the interval ≤ 10 m/s.

xx Δ values for very stable layering are uncertain.

/Legend/: (1) Type of layering; (2) Layering constant; (3) Wind speed v_x m/s; (4) Very stable.

APPENDIX 2

DATA MACHINE PROGRAM FOR CALCULATION OF GROUND COVERAGE FOR SPREAD OF SUBSTANCES IN LIQUID FORM

Engineer Arne Oehman

The program shown below was prepared at FOA Section 822 at the request of FOA 1. The program is coded in FORTRAN IV to be run through the FOA data processing machine IBM 7090/1401.

Basis, In and Out Values

The area of operation is a right angle coordinate system, in which x-y designates the ground plane with the x axis in the wind direction and z the height above the x-y plane.

At the moment of spraying the sprayed substance is in the form of a primary cloud build up of a number of cloud elements of the same shape, Each cloud element is in turn built of superimposed unit elements, with unit elements of the same type assumed to contain droplets of constant size.

The division into unit elements is described in the chapter SUBROUTINE MOFHST.

In values:

NRS	Operation No.	
SN	Layering type	
V5	Wind speed at ground level	m/s
NAMAL	Number of targets	
NEMT	Number of cloud elements	
EL	Cloud element side [1]	m
DST	Drop size interval in the unit element	μm
DMM	Mass median diameter	μm
W	Amount of substance/cloud element	g
XM, YM	Target coordinates	m
XO, YO, ZO	Cloud element coordinates	m
TS	Time to protection from target	s

1) The cloud element is assumed to be cube-formed.

Out Values:

Table of division into unit elements
Coverage on targets

Calculation Procedure:

1. Data entered

2. Median speed VM are calculated for all heights ZO

$$VM = \frac{1 - SN}{2} \cdot V5 \cdot \left(\frac{ZO}{5}\right)^{(SN/(2-SN))}$$

3. DELTA 2 = 3 · Δ^2 is calculated for all median wind speeds VM.

4. The cloud elements are divided into unit elements. Number of unit elements/cloud elements = NF. Speed of descent, VZ, and share of substance, DW for unit elements are determined. Substance density assumed to = 1.

5. For each target ground coverage at time TS is calculated through adding together coverages from the NEMT cloud elements. The contribution from a cloud element is in turn obtained by adding together the contributions from the NF unit elements in the cloud element. The total average for a target is:

$$BLG_{tot} = \sum_{J=1}^{NEMT} \sum_{K=1}^{NF} BLG\ (J,K)$$

in which BLG (J,K) is the contribution from the K unit element in the J cloud element.

The rest of the calculation procedure is apparent from the flow scheme.

SUBROUTINE MOFHST

Purpose of the Routine

The routine is employed for dividing a cloud element into unit elements and for calculating the speed of descent of the respective unit elements.

The division is made into intervals, determined by the parameter DST, and the speed of descent is calculated for the median value of the droplet size within the unit element.

Basis, In and Out Values

Call: CALL MOFHST (DST, DMM, I, DEL, VZ)

In values: Mass median diameter (DMM) μm
Interval width (DST) μm

Out values: Number of unit elements (I)
Partial volume for the various unit elements (DEL)
Speed of descent of the various unit elements (VZ) m/s

Calculations Procedure

The droplet size distribution in the cloud element is given by the equation

$$\frac{dV}{dX} = \frac{5^6 . V_0}{5! . D^6} . X^5 . \exp\left\{-\frac{5}{D} . X\right\}$$

in which X is the droplet diameter in μm
V_o is the total volume of substance in the cloud element
D = .877 · DMM

The value of the total volume is here unimportant and is set = 1

The equation is solved, giving:

$$V = \frac{5^6 \cdot V_0}{5! \cdot D^6} \cdot \int_0^{\infty} X^5 \cdot \exp\left\{-\frac{5}{D} \cdot X\right\} dX = V_0 \cdot \int_{\infty}^{0} \exp\left\{-\frac{5}{D} X\right\}$$

$$\sum_{j=0}^{5} \frac{1}{j!} \cdot \left(\frac{5}{D} \cdot X\right)^j$$

The speed of descent is determined by

$$VZ = k \cdot X^{(C1 + C2)} \cdot {}^{10}\log X + C3 \cdot ({}^{10}\log X)^2)$$

$$k = 3.0044 \cdot 10^{-5}$$

$$C1 = 2, \quad C2 = C2 = 0 \qquad 0 \leq X < 53 \ \mu m$$

$$C1 = 2.01302, \quad C2 = 0.12055, \quad C3 = -0.07422$$

$$\leq 53 \leq X < 800 \ \mu m$$

For I = 1, 2, 3 . . . . calculate

DEL(I) = part volume for the interval DST · I

VZ(I) = speed of descent for droplet size $X = \dfrac{DST \cdot (2 \cdot I - 1)}{2}$

Normally, the calculations are broken off when Σ part volumes > .999, whereupon the main program is returned to.

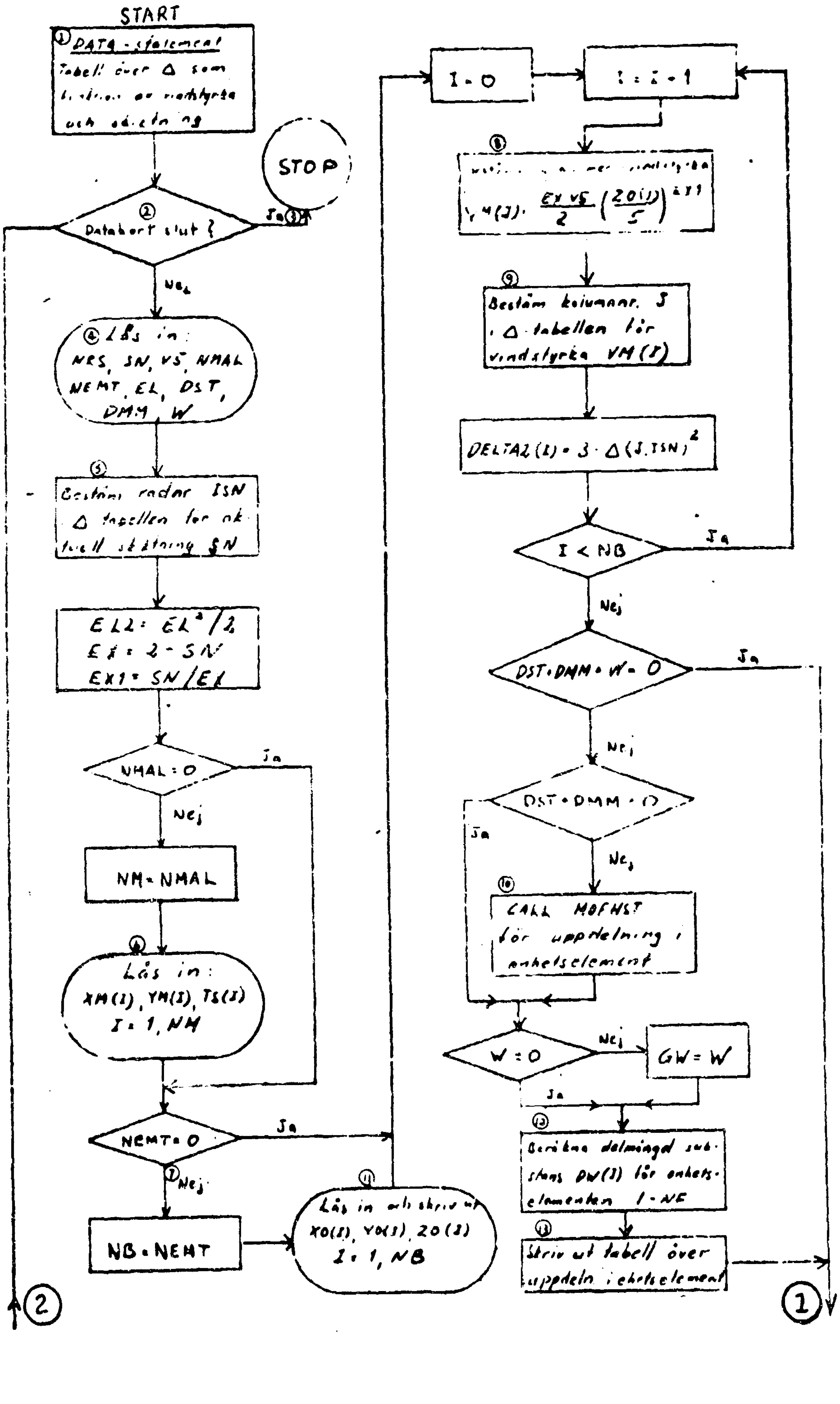
If this condition is not fulfilled, the calculations are broken off when the number of unit elements = 100, with the end line:

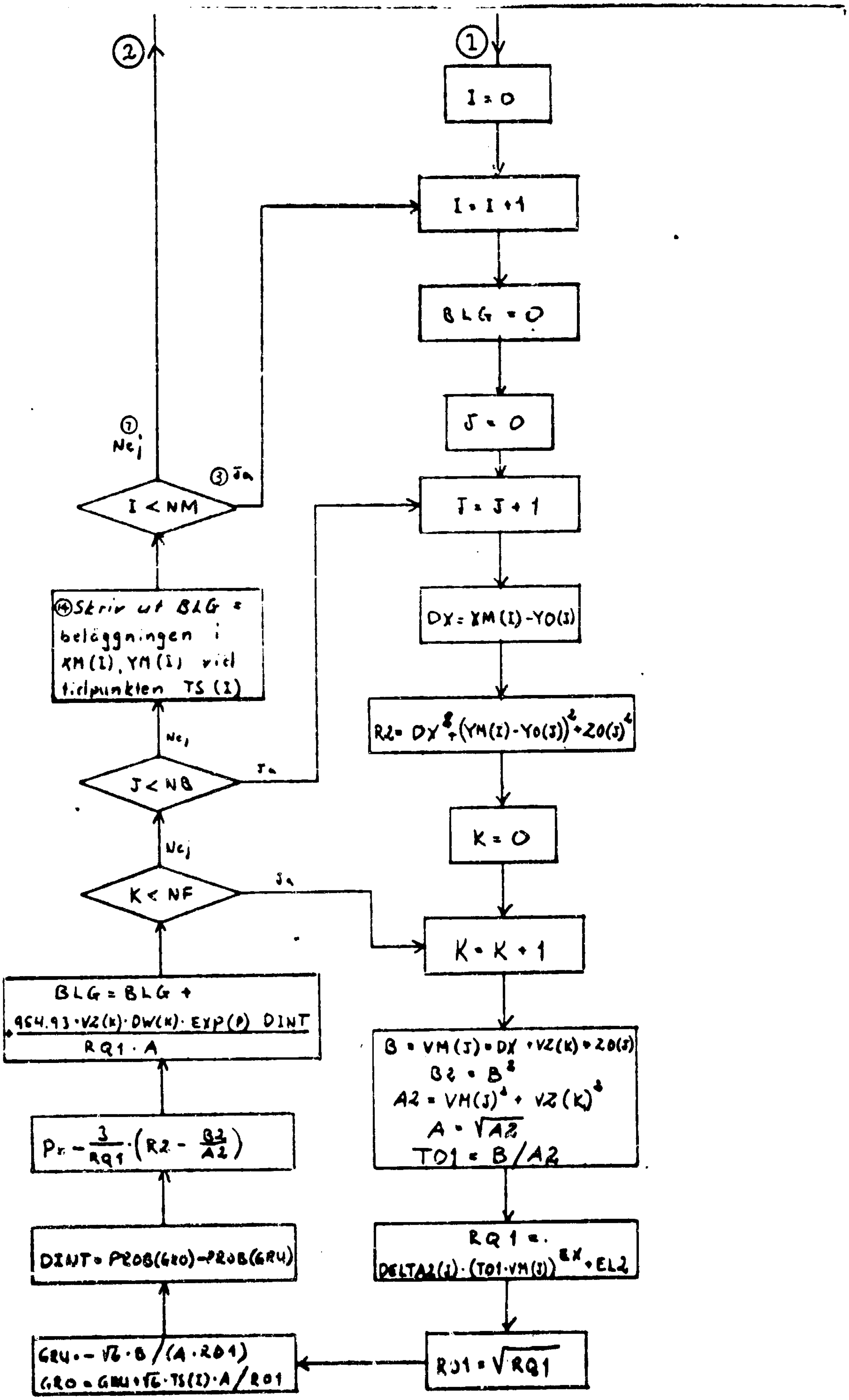
"NUMBER OF UNIT ELEMENTS = 100. REMAINING VOLUME XXX"

with return to the main program.

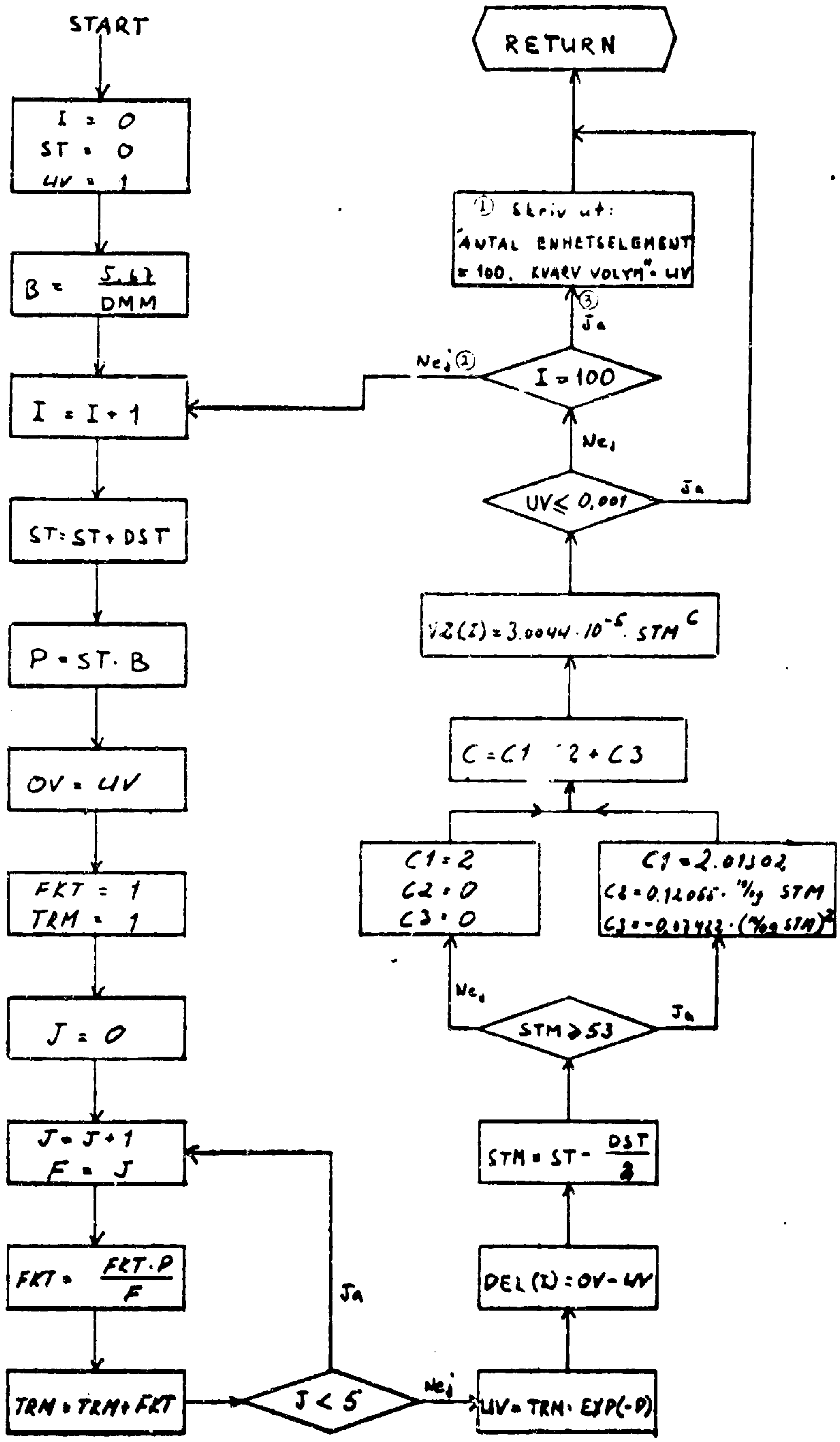
For a normal break off the width of the interval should be ≥10 μm.

Flow Scheme, Main Program





2
1
I = 0
I = I + 1
BLG = 0
J = 0
J = J + 1
DX = XM(I) - YO(J)
R2 = DX² + (YM(I) - YO(J))² + ZO(J)²
K = 0
K = K + 1
B = VM(J) · DX + VZ(K) · ZO(J)
B2 = B²
A2 = VM(J)² + VZ(K)²
A = √A2
TO1 = B / A2
RQ1 = DELTA2(J) · (TO1 · VM(J))^EX + EL2
RO1 = √RQ1
GRU = -√6 · B / (A · RO1)
GRO = GRU + √6 · TS(I) · A / RO1
DINT = PROB(GRO) - PROB(GRU)
P = -3/RQ1 · (R2 - B2/A2)
BLG = BLG + 954.93 · VZ(K) · DW(K) · EXP(P) · DINT / (RQ1 · A)
K < NF
Ja
Nej
J < NB
Ja
Nej
Skriv ut BLG = beläggningen i XM(I), YM(I) vid tidpunkten TS(I)
I < NM
Ja
Nej



START
I = 0
ST = 0
UV = 1
B = 5.67 / DMM
I = I + 1
ST = ST + DST
P = ST · B
OV = UV
FKT = 1
TRM = 1
J = 0
J = J + 1
F = J
FKT = FKT · P / F
TRM = TRM + FKT
J < 5
Ja
Nej
UV = TRM · EXP(-P)
DEL(I) = OV - UV
STM = ST - DST / 2
STM ≥ 53
Nej
Ja
C1 = 2
C2 = 0
C3 = 0
C1 = 2.01302
C2 = 0.12066 · ‰ STM
C3 = -0.07422 · (‰ STM)²
C = C1 2 + C3
VZ(I) = 3.0044 · 10⁻⁶ · STM^C
UV ≤ 0,001
Ja
Nej
I = 100
Nej ②
Ja ③
① Skriv ut:
"ANTAL ENHETSELEMENT = 100. KVARV VOLYM" = UV
RETURN

Flow Scheme, Main Program

/Legend/: (1) Table of Δ as a function of wind strength and layering; (2) End of data card?; (3) Yes; (4) Read in:; (5) Determine row No. ISN Δ table for layering; (6) Read in:; (7) No; (8) Determination of median wind strength; (9) Determine column No. J in Δ table for wind strength VM (I); (10) Call MOFHST for division into unit elements; (11) Read in and write out; (12) Estimate part amount substance DW (I) for unit element I-NF; (13) Write out table of division into unit elements; (14) Write out BLG = coverage in XM (I), YM (I) at time TS (I).

Flow Scheme MOFHST

/Legend/: (1) Write out: "Number of Unit Elements = 100. Remaining volume = LIV /?sic/; (2) No; (3) Yes.

- END -